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# Symmetry-deforming interactions of chiral $p$ -forms

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No-go theorems on gauge-symmetry-deforming interactions of chiral  $p$ -forms are reviewed. We consider the explicit case of  $p = 4$ ,  $D = 10$  and show that the only symmetry-deforming consistent vertex for a system of one chiral 4-form and two 2-forms is the one that occurs in the type  $II_B$  supergravity Lagrangian. Article based on a talk given by M. H. at the conference “Constrained Dynamics and Quantum Gravity 99” held in Villasimius (Sardinia), September 13-17 1999, to appear in the proceedings of the meeting (Nucl. Phys. B Proc. Suppl.).

## 1. INTRODUCTION

Chiral  $p$ -forms, which can be defined in  $(2p+2)$ -dimensional Minkowski spacetime for any even  $p$ , have the puzzling feature of not admitting a simple, manifestly covariant action principle even though their equations of motion are Lorentz-invariant, and even though one can define Poincaré generators that appropriately close according to the Poincaré algebra. The case  $p = 0$  (chiral bosons) arises in one formulation of the heterotic string, while the cases  $p = 2$  and  $p = 4$  are relevant to the  $M$ -theory 5-brane and type  $II_B$ -supergravity, respectively.

Although there is no simple, manifestly covariant action principle, there exists a non-manifestly covariant Lagrangian, which is quadratic in the fields in the free case and which yields the correct dynamics [1]. There exists also a non-polynomial, manifestly covariant action [2]. The two formulations are equivalent since one goes from the second one to the first one by appropriately gauge-fixing the auxiliary pure gauge field introduced in [2].

In a recent paper [3], we have shown that the

local interactions of chiral  $p$ -forms are severely constrained by the consistency requirement that they should not modify the number of physical degrees of freedom. More precisely, if one imposes that the deformed action (free action + interaction terms) be invariant under a deformed set of local gauge symmetries that continuously reduce to the gauge symmetries of the free theory as the coupling goes to zero, one finds in fact that no consistent interaction can deform the gauge symmetry at all. The gauge transformations must retain their original form under consistent deformations of the action. There is in particular no analog of the non-abelian Yang-Mills construction for chiral  $p$ -forms. The only available interactions must be invariant under the gauge symmetries of the abelian theory and leave the gauge structure untouched. This result generalizes to chiral  $p$ -forms the no-go theorems established for non-chiral  $p$ -forms in [4–8].

Motivated by the  $M$ -theory 5-brane, we considered in [3] the explicit case of  $p = 2$ . However, the argument is clearly quite general and applies to any  $p$ . Although we used as starting point the non-manifestly covariant Lagrangian of [1], the same obstructions would be present had we worked in the “PST formalism” of [2] because, as

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we have recalled, one recovers [1] from [2] by appropriately fixing the gauge. Furthermore, since the obstructions arise already at the level of the gauge symmetries (and not Lorentz invariance), they are easily detected in the non-manifestly covariant approach.

Every no-go theorem has of course the weaknesses of its hypotheses. In our case, these are:

1. The deformed action is local
2. The deformation is continuous
3. The system of local fields being dealt with is a system of  $N$  chiral  $p$ -forms described in the free limit by the non-covariant action of [1] (or the covariant action of [2], see above). Scalar fields or spinor fields are also allowed.

We stress, in particular, that no restriction on the number  $N$  of chiral  $p$ -forms was ever imposed. So, the  $p$ -forms can be labelled by any number of indices. Furthermore, no assumption on the order of the coupling vertices was made; these are not necessarily cubic, but can be quartic, quintic etc.

Accordingly, an interacting theory involving chiral  $p$ -forms with a deformed gauge symmetry must be either non-local (which is the most likely possibility in the M-theory 5-brane case, perhaps in the context of gerbes [9,10]), or non-perturbative and non-continuously connected with the free case, or have a richer field content.

Even this last possibility appears to be rather restrictive, however, and subject to strong no-go theorems that forbid a non-abelian deformation of the chiral  $p$ -form gauge symmetries analogous to the Yang-Mills deformation. One finds that the interactions may deform the gauge transformations but, when they do so, they cannot deform their algebra (to first-order in the deformation parameter). Furthermore, those that do deform the gauge transformations are few in number (if there is any at all). All other consistent interactions are again off-shell gauge-invariant under the abelian gauge symmetry and so do not deform it. We have not checked the rigidity of the gauge algebra in full generality in the chiral case, but the study of various explicit examples and the similarity with the non-chiral case where it has been

established [8] make us confident that this rigidity holds in the same manner in the chiral case. This leaves only the first two possibilities for non-abelian deformations.

We illustrate in these proceedings the no-go theorems on the symmetry-deforming interactions of exterior form gauge fields (involving chiral ones) by considering the system consisting of a chiral 4-form in ten spacetime dimensions together with  $n$  2-forms. This is relevant to type  $II_B$ -supergravity. We show that the only allowed symmetry-deforming interactions are in fact precisely those that appear in the supergravity Lagrangian. There are no others. These interactions deform the gauge transformations of the exterior forms but not their algebra, which remain abelian.

This case is a good example of the general cohomological techniques involved in the calculation of the deformations.

## 2. FIRST-ORDER DEFORMATIONS

### 2.1. Free action

We start with the free action

$$S[A_{ijkl}, B_{\lambda\mu}^a] = S_A + S_B \quad (1)$$

with

$$S_A = \int d^{10}x \left( \frac{1}{10} \varepsilon^{i_1 \dots i_5 j_1 \dots j_4} F_{i_1 \dots i_5} \dot{A}_{j_1 \dots j_4} - \frac{4!}{10} F^{i_1 \dots i_5} F_{i_1 \dots i_5} \right) \quad (2)$$

and

$$S_B = - \sum_a \frac{1}{2 \cdot 3!} \int d^{10}x H_{\lambda\mu\nu}^a H^{a\lambda\mu\nu}. \quad (3)$$

We have set

$$F_{ijklm} = 5 \partial_{[i} A_{jklm]}, \quad H_{\lambda\mu\nu}^a = 3 \partial_{[\lambda} B_{\mu\nu]}^a. \quad (4)$$

The PST manifestly covariant version of the theory may be found in [11,12]. We leave the number of 2-forms unspecified at this stage. We shall see that consistency to second-order forces the chiral 4-form to interact with only two 2-forms.

The action is invariant under the following gauge transformations

$$\delta_{\Lambda, \epsilon} A_{j_1 \dots j_4} = 4 \partial_{[j_1} \Lambda_{j_2 j_3 j_4]} \quad (5)$$

and

$$\delta_{\Lambda, \epsilon} B_{\lambda\mu}^a = \partial_{[\lambda} \epsilon_{\mu]}^a \quad (6)$$

The invariance of (3) and of the energy-term of (2) is obvious; the invariance of the kinetic term of (2) follows from an integration by parts and the Bianchi identity for the curvature  $F_{i_1 \dots i_5}$ . The kinetic term behaves thus like a Chern-Simons term.

The equations of motion that follow from (1) are the standard ones for the 2-forms and

$$4! \partial_m F^{mj_1 \dots j_4} - \varepsilon^{i_1 \dots i_5 j_1 \dots j_4} \partial_{[i_1} \dot{A}_{i_2 \dots i_5]} = 0 \quad (7)$$

for the chiral 4-form. From this equation, one derives

$$\dot{A}_{i_1 \dots i_4} - \frac{1}{5!} \varepsilon_{i_1 \dots i_4 j_1 \dots j_5} F^{j_1 \dots j_5} = 4 \partial_{[i_1} u_{i_2 i_3 i_4]} \quad (8)$$

(assuming the 4th Betti number of the spatial sections to vanish). This is the self-duality condition  $F = *F$  if one identifies the arbitrary functions  $u_{i_2 i_3 i_4}$  with the gauge components  $A_{0 i_2 i_3 i_4}$ .

A chiral 4-form and two 2-forms are present in the spectrum of type  $II_B$  supergravity in 10 dimensions [13–15]. The 4-form signals the presence of the D3 brane and the two 2-forms correspond to the fundamental string and the D1-brane, respectively.

## 2.2. BRST differential

We shall here just outline the general method and ideas without going into the details. These will be given in [16].

To determine the possible consistent interactions, we follow the method of [17]. The question boils down to computing the BRST cohomological group  $H^0(s|d)$  at ghost number zero, i.e., one must find the general solution of the cocycle condition

$$sa + db = 0, \quad gh(a) = 0 \quad (9)$$

modulo trivial ones (of the form  $sm + dn$ ). Here  $s$  is the BRST differential and  $d$  the spacetime exterior derivative.

The solutions of (9) are expanded according to the antighost (or antifield) number,

$$a = a_0 + a_1 + \dots + a_k. \quad (10)$$

The fact that the expansion stops at a finite order  $k$  follows from the fact  $a_0$  contains only a finite number of derivatives, which already excludes a weak form of non-locality.

Each term in the expansion (10) has total ghost number zero. The first term does not involve the ghosts or the antifields. The next terms depend both on the antifields and the ghosts, but in such a way that the antighost number exactly balances the pure ghost number (the higher  $i$ , the “more”  $a_i$  involves antifields and hence also ghosts). The antifield-independent term  $a_0$  defines the first-order consistent vertex. The antifield-dependent terms define the deformation of the gauge-symmetry and its algebra. In particular, the term  $a_1$ , which is linear in the antifields conjugate to the classical fields, defines the deformation of the gauge transformations. Similarly, the term in  $a_2$  which is linear in the antifields conjugate to the first ghosts and quadratic in those ghosts, corresponds to the deformation of the gauge algebra. If this term is absent, the gauge algebra remains abelian (to first order) even after the interaction is switched on. This occurs if the most general solution of (9) is at most linear in the ghosts, up to trivial terms that can be absorbed through redefinitions. This is what happens for the model at hand.

The BRST differential is explicitly given by  $s = \delta + \gamma$  where one has, in the 4-form sector with ghosts  $C_{ijk}$ , ghosts of ghosts  $D_{ij}$ ,  $E_i$  and  $\eta$ , and antifields  $A^{*ijkl}$ ,  $C^{*ijk}$ ,  $D^{*ij}$ ,  $E^{*i}$ ,  $\eta^*$ ,

$$\gamma A_{ijkl} = 4\partial_{[i} C_{jkl]}, \quad \delta A_{ijkl} = 0 \quad (11)$$

$$\gamma C_{ijk} = 3\partial_{[i} D_{jk]}, \quad \delta C_{ijk} = 0 \quad (12)$$

$$\gamma D_{ij} = 2\partial_{[i} E_{j]}, \quad \delta D_{ij} = 0 \quad (13)$$

$$\gamma E_i = \partial_i \eta, \quad \delta E_i = 0 \quad (14)$$

$$\gamma \eta = 0, \quad \delta \eta = 0 \quad (15)$$

and

$$\delta A^{*ijkl} = 4! \partial_m F^{mijkl} - \varepsilon^{ijklm_1 \dots m_5} \partial_{[m_1} \dot{A}_{m_2 \dots m_5]}, \quad (16)$$

$$\gamma A^{*ijkl} = 0 \quad (17)$$

$$\delta C^{*ijk} = \partial_m A^{*mijk}, \quad \gamma C^{*ijk} = 0, \quad (18)$$

$$\delta D^{*ij} = \partial_m C^{*mij}, \quad \gamma D^{*ij} = 0, \quad (19)$$

$$\delta E^{*i} = \partial_m D^{*mi}, \quad \gamma E^{*i} = 0, \quad (20)$$

$$\delta\eta^* = \partial_m E^{*m}, \quad \gamma\eta^* = 0. \quad (21)$$

The ghosts  $C_{ijk}$  and ghosts of ghosts  $D_{ij}$ ,  $E_i$  and  $\eta$  have respectively pure ghost number 1, 2, 3 and 4. The antifields  $A^{*ijkl}$ ,  $C^{*ijk}$ ,  $D^{*ij}$ ,  $E^{*i}$  and  $\eta^*$  have respectively antighost number 1, 2, 3, 4 and 5.

In the 2-form sector, the differentials  $\delta$  and  $\gamma$  are given by

$$\gamma B_{\lambda\mu}^a = \partial_\lambda \xi_\mu^a - \partial_\mu \xi_\lambda^a, \quad \delta B_{\lambda\mu}^a = 0, \quad (22)$$

$$\gamma \xi_\lambda^a = \partial_\lambda \sigma^a, \quad \delta \xi_\lambda^a = 0, \quad (23)$$

$$\gamma \sigma^a = 0, \quad \delta \sigma^a = 0, \quad (24)$$

$$\gamma B_a^{*\lambda\mu} = 0, \quad \delta B_a^{*\lambda\mu} = \partial_\rho H_a^{\rho\lambda\mu}, \quad (25)$$

$$\gamma \xi_a^{*\lambda} = 0, \quad \delta \xi_a^{*\lambda} = \partial_\rho B_a^{*\rho\lambda}, \quad (26)$$

$$\gamma \sigma_a^* = 0, \quad \delta \sigma_a^* = \partial_\rho \xi_a^{*\rho}. \quad (27)$$

The ghosts  $\xi_\lambda^a$  have pure ghost number 1 and the ghosts of ghosts  $\sigma^a$  have pure ghost number 2. The antifields  $B_a^{*\lambda\mu}$ ,  $\xi_a^{*\lambda}$  and  $\sigma_a^*$  have respectively antighost number 1, 2 and 3.

The (total) ghost number  $gh$  is the difference between the pure ghost number and the antighost number. The component  $\delta$  and  $\gamma$  of  $s$  have (pureghost, antighost) number equal to (0,-1) and (1,0) respectively. The equation  $s^2 = 0$  is equivalent to  $\delta^2 = \delta\gamma + \gamma\delta = \gamma^2 = 0$ .

### 2.3. Deformations

We assume  $k > 0$  in (10) since when  $a$  reduces to  $a_0$ , it does not modify the gauge transformations and there is nothing to be demonstrated. The idea of the proof is to show that the building blocks out of which  $a_k$  can be made for  $k > 0$ , have ghost numbers that cannot generically add up to zero. Thus, there is no or very few ways to write down an acceptable  $a_k$  for  $k > 0$ .

The building blocks of  $a_k$  are, on the one hand, the polynomials in the last ghosts of ghosts  $\sigma^a$  and  $\eta$  and their temporal derivatives  $\partial_0\eta$ ,  $\partial_{00}\eta$ , etc, and, on the other hand, the elements of the “invariant characteristic cohomology”  $H_k(\delta|d)$  or  $H(\delta|\tilde{d})$ , where  $\tilde{d}$  is the spatial exterior derivative  $dx^i\partial_i$ . The former have even ghost number, while the latter generically have odd antighost number. So, their products generically fail to have total ghost number zero, with only one exception for the system under study.

To see this, one follows the argument given in [3] and one finds that the last term  $a_k$  in the general solution of the cocycle condition (9) can be assumed to be annihilated by  $\gamma$ ,  $\gamma a_k = 0$ . Thus, it reads

$$a_k = \sum_I P^I \omega^I \quad (28)$$

(up to trivial terms), where  $P^I$  is a polynomial in the curvatures  $F_{ijklm}$  and  $H_{\lambda\mu\nu}^a$ , in the antifields, and in their spacetime derivatives, while the  $\omega^I$  form a basis in the algebra of polynomials in the variables  $\sigma^a$ ,  $\eta$ ,  $\partial_0\eta$ ,  $\partial_{00}\eta$ , etc, which are the generators of the cohomology  $H(\gamma)$  in positive ghost number [the fact that the time derivative of eta is a generator while the time derivative of sigma isn't is a consequence from the fact that the 4-form is treated non-covariantly while the 2 form is treated covariantly].

The coefficient  $P^I$  of  $\omega^I$  in the sum (28), which is a 10-form, is subject to different conditions depending on to whether the element  $\omega^I$  that it multiplies depends on  $\eta$  and its time derivatives or not.

1. If  $\omega^I$  involves at least one time derivatives of  $\eta$  (including  $\partial_0^0\eta \equiv \eta$ ), then the corresponding coefficient  $P^I$  must be of the form  $P^I = Q^I dx^0$  where  $Q^I$  is a spatial 9-form solution of  $\delta Q^I + \tilde{d}N^I = 0$ . Here,  $\tilde{d}$  is the spatial exterior derivative,  $\tilde{d}a = dx^k\partial_k a$ . However, there is no non-trivial solution to that equation that would match the ghost number of  $\omega^I$ , which is an even number  $\geq 4$  since  $\omega^I$  contains at least one  $\eta$  or one of its time derivatives. The only non-trivial solutions are indeed in antighost number 5, 2 and 1. Thus, in this case, one cannot construct an  $a_k$  ( $k > 0$ ) with total ghost number zero. This is exactly the situation described in [3] - and is the only case to be considered for a pure system of chiral  $p$ -forms. Note the similarity with the Hamiltonian analysis of consistent couplings done in [18].
2. If  $\omega^I$  depends only on  $\sigma^a$ , the corresponding coefficient  $P^I$  must be a 10-form that solves

the equation  $\delta P^I + dN^I = 0$ . It must be of even antighost number to match the even ghost number of  $\omega^I$ . There is one possibility, namely,  $C^* \wedge H^a$ , where  $C^*$  is the 7-form  $\varepsilon_{0i_1 \dots i_6 j_1 j_2 j_3} C^{*j_1 j_2 j_3} dx^0 \wedge dx^{i_1} \wedge \dots \wedge dx^{i_6}$ . It has antighost number 2 and can be combined with  $\sigma^a$  to yield a non-trivial  $a_2$ . Specifically - and in density notations -,

$$a_2 = C^{*i_1 i_2 i_3} H_{i_1 i_2 i_3}^a \sigma^b \mu_{ab} \quad (29)$$

where  $\mu_{ab}$  is a matrix of coupling constants having dimension (length)<sup>4</sup>. The matrix  $\mu_{ab}$  must be antisymmetric since otherwise  $H_{i_1 i_2 i_3}^a \sigma^b \mu_{ab} = \gamma(\text{something}) + d(\text{something})$  and  $a_2$  can be removed. This possibility specifically requires the presence of the 2-forms and is not available for a pure system of chiral forms. Besides  $C^* \wedge H^a$ , there is no other non trivial class in  $H_k(\delta|d)$  with even antighost number.

Since the above  $a_2$  is the only possibility, we carry on the discussion with it. The equation  $\delta a_2 + \gamma a_1 + db_1 = 0$  determines  $a_1$  up to a trivial term,

$$a_1 = -A^{*m i_1 i_2 i_3} H_{i_1 i_2 i_3}^a \xi_m^b \mu_{ab}. \quad (30)$$

The corresponding interaction vertex  $a_0$  is then found to be

$$\begin{aligned} a_0 = & -12 F^{k m i_1 i_2 i_3} H_{i_1 i_2 i_3}^a B_{k m}^b \mu_{ab} \\ & - \frac{3}{10} \varepsilon^{i_0 \dots i_3 l_1 \dots l_5} F_{l_1 \dots l_5} H_{i_0 i_2 i_3}^a B_{i_1 i_0}^b \mu_{ab} \\ & + \frac{1}{5} \varepsilon^{i_0 \dots i_3 l_1 \dots l_5} F_{l_1 \dots l_5} H_{i_1 i_2 i_3}^a B_{0 i_0}^b \mu_{ab} \end{aligned} \quad (31)$$

The term  $a_0$  is determined up to vertices that are gauge-invariant under the abelian gauge-symmetry (possibly modulo a total derivative). Since these do not modify the gauge transformations, we do not include them in the sequel and stick to the choice (31) for  $a_0$ . This choice has the smallest number of derivatives.

### 3. COMPLETE ACTION

#### 3.1. New gauge symmetries

Once the first-order vertex is determined, one can analyse the conditions imposed by consistency to second-order. These conditions have

also a cohomological interpretation [17]. We shall complete the action in the explicit case where the first-order vertex is solely given by the above deformation  $a_0$ , without extra gauge-invariant terms.

To discuss the consistency conditions at second-order, we first note that the gauge transformation of the field  $A_{ijkl}$  is modified by the interaction since  $a$  contains a piece linear in  $A^{*m i_1 i_2 i_3}$ , from which one reads the new gauge transformations

$$\delta_{\Lambda, \epsilon} A_{ijkl} = 4 \partial_{[i} \Lambda_{jkl]} + \epsilon_{[i}^a H_{jkl]}^b \mu_{ab} \quad (32)$$

The term  $a_2$ , linear in the antifield  $C^{*ijk}$ , signals the corresponding modification of the reducibility identity. The gauge transformations of  $B_{\mu\nu}^a$  are unchanged since there is no term proportional to  $B_a^{*\mu\nu}$  in  $a_1$ .

#### 3.2. Action

To derive the complete action, it is convenient to introduce the redefined field strength

$$\mathcal{F}_{i_1 \dots i_5} = F_{i_1 \dots i_5} - \frac{5}{2} B_{[i_1 i_2}^c H_{i_3 i_4 i_5]}^d \mu_{cd} \quad (33)$$

which has the property of being invariant under the transformation (32). Similarly, one defines

$$\begin{aligned} \mathcal{D}_0 A_{j_1 \dots j_4} = & \dot{A}_{j_1 \dots j_4} - \frac{3}{2} B_{[j_1 j_2}^a H_{0 j_3 j_4]}^b \mu_{ab} \\ & - B_{0[j_1}^a H_{j_2 j_3 j_4]}^b \mu_{ab} \end{aligned} \quad (34)$$

which transforms into spatial total derivative,  $\delta_{\Lambda, \epsilon} \mathcal{D}_0 A_{j_1 \dots j_4} \sim \partial_{[j_1} f_{j_2 j_3 j_4]}$ .

By following the recursive construction of [17], one finds that  $O(\mu^2)$ -terms must be added to the action. There is no modification to the gauge transformations at that order, which remain given by (32) and  $\delta_{\Lambda, \epsilon} B^a = d\epsilon^a$ . Rather than following in detail the recursive construction of [17], we shall give the final answer. The complete action takes the form

$$\tilde{S}[A_{ijkl}, B_{\lambda\mu}^a] = \tilde{S}_A + S_B, \quad (35)$$

with  $\tilde{S}_A$  given by

$$\tilde{S}_A = \int d^{10}x \left( \frac{1}{10} \varepsilon^{i_1 \dots i_5 j_1 \dots j_4} \mathcal{F}_{i_1 \dots i_5} \mathcal{D}_0 A_{j_1 \dots j_4} \right)$$

$$\begin{aligned}
& -\frac{1}{2}\varepsilon^{i_1\dots i_5 j_1\dots j_4} H_{0i_1 i_2}^a H_{i_3 i_4 i_5}^b A_{j_1\dots j_4} \mu_{ab} \\
& -\frac{4!}{10} \mathcal{F}^{i_1\dots i_5} \mathcal{F}_{i_1\dots i_5}
\end{aligned} \tag{36}$$

and  $S_B$  unchanged.

One easily verifies that the complete action is invariant under the new gauge symmetries provided the 10-form

$$\epsilon^a \wedge H^b \mu_{ab} \wedge H^c \wedge H^d \mu_{cd} \tag{37}$$

vanishes. This condition will be fulfilled only if the antisymmetric matrix  $\mu_{ab}$  is of rank two. This is most easily seen by bringing  $\mu_{ab}$  to canonical form,

$$\mu_{ab} = \begin{pmatrix} 0 & \alpha & 0 & 0 & \dots & 0 & 0 \\ -\alpha & 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & 0 & \beta & \dots & 0 & 0 \\ 0 & 0 & -\beta & 0 & \dots & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \dots & \cdot & \cdot \\ 0 & 0 & 0 & 0 & \dots & 0 & \lambda \\ 0 & 0 & 0 & 0 & \dots & -\lambda & 0 \end{pmatrix} \tag{38}$$

where we have assumed an even number of 2-forms. [In the case of an odd number of 2 forms, there would be an extra row and an extra column of zeros, making it clear that the “last” 2-form decouples.] Only one eigenvalue can be non-zero in (38): if two are non-zero, say  $\alpha$  and  $\beta$ , then the sum (37) contains the term  $\alpha\beta\epsilon^1 \wedge H^2 \wedge H^3 \wedge H^4$ , which is non-zero. Thus we see that consistency to second order imposes that only two 2-forms couple in fact to the chiral 4-form. [The term (37), with  $\epsilon^a$  replaced by the ghost  $\xi^a$ , is a non-trivial solution of  $sa + db$  and so represents a true obstruction, unless it is zero.]

The action (35) with two 2-forms is the action for the bosonic sector of type  $II_B$ -supergravity (with scalar fields set equal to zero and the metric equal to  $\eta_{\mu\nu}$ ).

### 3.3. Equations of motion

The equations of motion that follow from varying the complete action with respect to  $A_{ijkl}$  are  $4! \partial_m \mathcal{F}^{mijkl} - \varepsilon^{m_1\dots m_5 i j k l} \partial_{[m_1} \mathcal{D}_0 A_{m_2\dots m_5]} = 0$  (39)

Assuming as before that the 4th Betti number of the spatial sections is zero, one gets

$$\mathcal{D}_0 A_{ijkl} - \frac{1}{5!} \varepsilon_{ijklm_1\dots m_5} \mathcal{F}^{m_1\dots m_5} = 4 \partial_{[i} u_{jkl]} \tag{40}$$

Defining  $A_{0ijk}$  to be  $u_{ijk}$ , one can rewrite (40) as

$$\mathcal{F} = {}^* \mathcal{F} \tag{41}$$

where

$$\mathcal{F} = dA - \frac{5}{2} \mu_{ab} B^a \wedge H^b. \tag{42}$$

These are the modified chirality conditions in covariant form.

The equations of motion for the 2-forms are, on the other hand

$$d^* H^a + 3! 5! \mu_{ab} H^b \mathcal{F} = 0. \tag{43}$$

Note that one could have derived the action (35) by starting from the action describing a non-chiral 4-form interacting with two 2-forms through Chapline-Manton and Chern-Simons couplings with coefficients such that  $d\mathcal{F} = d^* \mathcal{F}$  and making the projection to the chiral sector through constraints.

### 3.4. Algebra of new gauge transformations

It is easy to verify that the algebra of the new gauge transformations is abelian. This is because the new terms involve only the curvatures and no bare  $A$  or  $B^a$ . Thus, the gauge algebra is not deformed.

## 4. CONCLUSIONS

In this paper, we have studied the interactions for a mixed system containing a chiral 4-form and  $n$  2-forms in 10 dimensions. We have shown the uniqueness of the supergravity vertices. Any interaction that deforms the gauge transformations necessarily invoke them. The gauge algebra is, however, untouched. We stress that this result has been obtained without invoking supersymmetry. Our analysis illustrates the rigidity of  $p$ -form systems in the local field-theoretical context, already well appreciated in the non-chiral case.

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